

## A Proof of Lemma 1

We show that  $\zeta^{\mathcal{L}}(s, a; D_{s,a}, k)$  defined in Eq. (7) decays to 0 at a rate of  $1/n_{s,a}^2$ . We present the proof for  $k = 1$ . The extension to  $k > 1$  is straightforward.

In finite MDPs, we have to learn separate transition probability tables  $\mathcal{T}(s' | s, a)$  for each  $s, a$ . For simplicity, we focus on one fixed  $(s, a)$  and investigate how to estimate the distribution  $\mathcal{T}(s')$ . We consider a Dirichlet learner where  $\alpha$  denotes the Dirichlet posterior based on the data  $D_{s,a} = \{s'_i\}_{i=1}^{n_{s,a}}$  and  $\alpha'$  be the posterior based on the reduced data set  $D_{s,a}^{k=-1}$ , that is,  $\alpha'_c + 1 = \alpha_c$  where  $c$  is the outcome of the missing experience in  $D_{s,a}^{k=-1}$ , and  $\alpha'_j = \alpha_j$  for all  $j \neq c$ . Given Dirichlet parameters  $\alpha$ , the MAP model  $\hat{\mathcal{T}}_\alpha(s')$  is given by the vector  $\alpha/\bar{\alpha}$ ,  $\bar{\alpha} = \sum_i \alpha_i$ , and we estimate  $\zeta$  using Eq. (7). The log-likelihood of the data  $D_{s,a}$  under  $\hat{\mathcal{T}}_\alpha$  is

$$\begin{aligned} L^+ &:= \log P(D_{s,a} | \hat{\mathcal{T}}_\alpha) \\ &= \log \prod_{i=1}^{n_{s,a}} \frac{\alpha_{s'_i}}{\bar{\alpha}} = \sum_{i=1}^{n_{s,a}} \log \alpha_{s'_i} - n_{s,a} \log \bar{\alpha} \end{aligned} \quad (14)$$

The likelihood of the data  $D_{s,a}$  under  $\hat{\mathcal{T}}_{\alpha'}$  is

$$\begin{aligned} L^- &:= \log P(D_{s,a} | \hat{\mathcal{T}}_{\alpha'}) \\ &= \log \left( \prod_{i=1, s'_i \neq c}^{n_{s,a}} \frac{\alpha_{s'_i}}{\bar{\alpha} - 1} \right) \left( \prod_{i=1, s'_i = c}^{n_{s,a}} \frac{\alpha_c - 1}{\bar{\alpha} - 1} \right) \\ &= \sum_{i=1, s'_i \neq c}^{n_{s,a}} \log \alpha_{s'_i} + \sum_{i=1, s'_i = c}^{n_{s,a}} \log(\alpha_c - 1) - n_{s,a} \log(\bar{\alpha} - 1) \end{aligned} \quad (15)$$

The average difference is

$$\begin{aligned} \zeta^{\mathcal{L}}(s, a; k = 1) &= \frac{1}{n_{s,a}} |L^+ - L^-| \\ &= \frac{1}{n_{s,a}} \sum_{i=1, s'_i = c}^{n_{s,a}} (\log \alpha_c - \log(\alpha_c - 1)) - \log \bar{\alpha} + \log(\bar{\alpha} - 1) \\ &= \frac{n_{s,a,c}}{n_{s,a}} \log \frac{1}{1 - \frac{1}{\alpha_c}} + \log(1 - \frac{1}{\bar{\alpha}}). \end{aligned} \quad (16)$$

Since  $\bar{\alpha} \propto n_{s,a}$ , by taking the derivative of the expected value of  $\zeta^{\mathcal{L}}(s, a; k = 1)$  we can verify that  $E_{D_{s,a}}(\zeta(s, a; D_{s,a}, k = 1)) = O\left(\frac{1}{n_{s,a}^2}\right)$ .  $\square$